Q1a

1a

(a) Rewrite $\tan\theta \operatorname{cosec}\theta$ as a single trigonometric function. [2]

(b) Hence solve, in the range $-\pi < \theta \le \pi$, the equation

$$\tan\theta \operatorname{cosec}\theta = -\frac{2\sqrt{3}}{3}.$$ [3]

a) $\operatorname{Tan}\theta = \frac{\operatorname{Sin}\theta}{\operatorname{Cos}\theta}$ $\quad \operatorname{Cosec}\theta = \frac{1}{\operatorname{Sin}\theta}$

$$\frac{\cancel{\operatorname{Sin}\theta}}{\operatorname{Cos}\theta} \cdot \frac{1}{\cancel{\operatorname{Sin}\theta}} = \frac{1}{\operatorname{Cos}\theta}$$

$$\frac{1}{\operatorname{Cos}\theta} = \operatorname{Sec}\theta$$

$$\boxed{\operatorname{Tan}\theta \operatorname{Cosec}\theta = \operatorname{Sec}\theta}$$

Q1b

1b

(a) Rewrite $\tan\theta \operatorname{cosec}\theta$ as a single trigonometric function. [2]

(b) Hence solve, in the range $-\pi < \theta \le \pi$, the equation

$$\tan\theta \operatorname{cosec}\theta = -\frac{2\sqrt{3}}{3}.$$ [3]

b) $\operatorname{Tan}\theta \operatorname{Cosec}\theta = \operatorname{Sec}\theta$

$$\operatorname{Sec}\theta = \frac{1}{\operatorname{Cos}\theta}$$

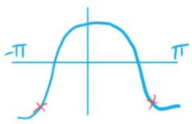
$$\frac{1}{\operatorname{Cos}\theta} = \frac{-2\sqrt{3}}{3}$$

$$\operatorname{Cos}\theta = \frac{3}{-2\sqrt{3}}$$

RATIONALISE

$$\operatorname{Cos}\theta = -\frac{\sqrt{3}}{2}$$

$$\operatorname{Cos}^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5}{6}\pi$$



$$\boxed{\theta = -\frac{5}{6}\pi, \frac{5}{6}\pi}$$

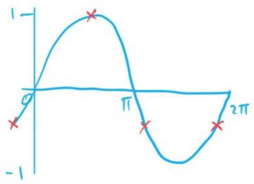
Q2

2

Solve, in the range $0 \le \theta \le 2\pi$, the equation

$$\frac{2}{\operatorname{cosec}\theta} - \operatorname{cosec}\theta = 1.$$

[6]



MULTIPLY FACTORISE SOLVE

$$2 - \operatorname{cosec}^2\theta = \operatorname{cosec}\theta$$

$$\operatorname{cosec}^2\theta + \operatorname{cosec}\theta - 2 = 0$$

$$(\operatorname{cosec}\theta + 2)(\operatorname{cosec}\theta - 1) = 0$$

$$\frac{1}{\sin\theta} + 2 = 0 \qquad \frac{1}{\sin\theta} - 1 = 0$$

$$\frac{1}{\sin\theta} = -2 \qquad \frac{1}{\sin\theta} = 1$$

$$\sin\theta = -\frac{1}{2} \qquad \sin\theta = 1$$

$$\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{1}{6}\pi \qquad \sin^{-1}(1) = \frac{1}{2}\pi$$

$$\pi + \frac{1}{6}\pi = \frac{7}{6}\pi$$

$$2\pi - \frac{1}{6}\pi = \frac{11}{6}\pi$$

$$\boxed{\theta = \frac{1}{2}\pi, \frac{7}{6}\pi, \frac{11}{6}\pi}$$

Q3

3

Using the double angle formula $\sin 2A \equiv 2\sin A\cos A$, find the solutions to the equation

$$\sec x \operatorname{cosec} x - 75 = 5\operatorname{cosec} 2x$$

in the range $-\pi < x \le \pi$. Give your answers correct to 3 significant figures.

[6]

$$-2\pi < 2x \le 2\pi$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\frac{1}{\cos x\sin x} \times \frac{2}{2} = \frac{2}{2\sin x\cos x} = \frac{2}{\sin 2x}$$

$$\frac{2}{\sin 2x} - 75 = \frac{5}{\sin 2x}$$

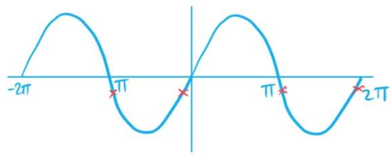
$$-75 = \frac{3}{\sin 2x}$$

$$\sin 2x = \frac{\cancel{3}^{1}}{-\cancel{75}_{25}} = -\frac{1}{25}$$

LET $z = 2x$ TRANSFORM THE RANGE

$$\sin z = -\frac{1}{25}$$

$$\sin^{-1}\left(-\frac{1}{25}\right) = -0.04001\ldots$$



$z = -0.04, -3.10, 3.18, 6.24$

$-\pi + 0.04 \quad \pi + 0.04 \quad 2\pi - 0.04$

$\div 2$

$$\boxed{x = -1.55, -0.020, 1.59, 3.12 \ (3sf)}$$

Q4a

4a

(a) Show that the equation

$$2\cot^2 x = 1 - 5\operatorname{cosec} x$$

can be rewritten in the form

$$(2\operatorname{cosec} x - 1)(\operatorname{cosec} x + 3) = 0.$$

[3]

(b) Hence solve, in the range $0 \le x \le 2\pi$, the equation

$$2\cot^2 x = 1 - 5\operatorname{cosec} x$$

giving your answers correct to 3 significant figures.

[3]

a) $1 + \cot^2 x \equiv \operatorname{cosec}^2 x$

$\cot^2 x = \operatorname{cosec}^2 x - 1$

$2(\operatorname{cosec}^2 x - 1) = 1 - 5\operatorname{cosec} x$

$2\operatorname{cosec}^2 x - 2 = 1 - 5\operatorname{cosec} x$

$2\operatorname{cosec}^2 x + 5\operatorname{cosec} x - 3 = 0$

$(2\operatorname{cosec} x - 1)(\operatorname{cosec} x + 3) = 0$

Q4b

4b

(a) Show that the equation

$$2\cot^2 x = 1 - 5\operatorname{cosec} x$$

can be rewritten in the form

$$(2\operatorname{cosec} x - 1)(\operatorname{cosec} x + 3) = 0.$$

[3]

(b) Hence solve, in the range $0 \le x \le 2\pi$, the equation

$$2\cot^2 x = 1 - 5\operatorname{cosec} x$$

giving your answers correct to 3 significant figures.

[3]

b) $(2\operatorname{cosec} x - 1)(\operatorname{cosec} x + 3) = 0$

$2\operatorname{cosec} x - 1 = 0$ $\quad$ $\operatorname{cosec} x + 3 = 0$

$\operatorname{cosec} x = \frac{1}{2}$ $\quad$ $\operatorname{cosec} x = -3$

$\frac{1}{\sin x} = \frac{1}{2}$ $\quad$ $\frac{1}{\sin x} = -3$

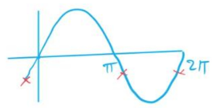
$\sin x = 2$ $\quad$ $\sin x = -\frac{1}{3}$

DOESNT EXIST

$\operatorname{cosec}\theta = k$ NO SOLUTIONS

$-1 < k < 1$

$\sin^{-1}\left(-\frac{1}{3}\right) = -0.339\ldots$



$\pi + x$ $\quad$ $2\pi - x$

$3.4814\ldots$ $\quad$ $5.9433\ldots$

$\boxed{x = 3.48,\ 5.94 \ (3sf)}$

Q5

5

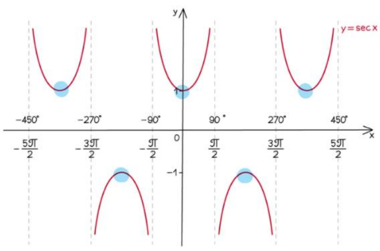
(i) Sketch, in the interval $-2\pi \le \theta \le 2\pi$, the graph of $y = -5 + \frac{1}{2}\sec\theta$, include asymptotes and label the coordinates of all maximum and minimum points.

(ii) Hence deduce the range of values for k for which the equation $-5 + \frac{1}{2}\sec\theta = k$ has no solutions.

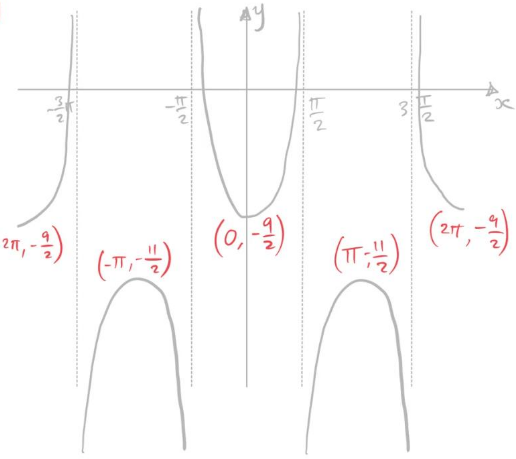
[5]

VERTICAL STRETCH SF $\frac{1}{2}$

TRANSLATION $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$



i)



ii) $\boxed{-5.5 < k < -4.5}$